Multi-phase Trajectory Optimization for Airliners' Formation Flight Considering TCAS Constraints

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***Abstract*—With the current Covid-19 pandemic and the resulting substantial decrease in aerial transportation, many airlines have reported heavy financial losses. This work is an attempt to demonstrate how "formation flights" could be helpful to decrease airlines' operating costs. The proposed approach allows airlines to operate with lower than usual "passenger load-factors". In this work, we show how "Extended formation flights" could help reduce airliners operating costs through decreasing aircrafts' "mission fuel consumption". In this line of thought, two distinctive but interconnected problems have been investigated in the process of creating and maintaining a successful formation flight. In this work, we show how any two aircraft could successfully share the benefit of drag-reduction. The methodology could easily be expanded to formations with three or more aircraft. In the mathematical model, we describe how two aircraft with similar size, but with different flight origins and different destinations could cooperate throughout the cruise-portion of their flight to decrease their cruising drag. In fact, we allow each aircraft to periodically fly into wing-vortex of one another. We have also, incorporated the role of existing Traffic Collision Avoidance System (TCAS) limitations into the model. To investigate different flying scenarios, an in-house computer program with the help of the MATLAB environment has been developed that modifies each individual aircraft's induced drag based on their relative positions. The elaborate mathematical model with the help of ADS-B system, allows each aircraft to change positions as needed. The current approach allows aircraft participating in a formation to enjoy the benefits during the flight as it allows aircraft to rotate positions as many times as needed. So, no further book-keeping is required for formation flights. Further studies might be required for detail of the rotation process. Different case-studies reveal that TCAS limitations has considerable effects on maintaining a rendezvous. We further propose a new class of TCAS that differentiates those flights which are in formation from those that are not.**

***Index Terms*—Extended formation flight, TCAS, Traffic collision avoidance system, Airborne collision avoidance system, Direct collocation methods, Optimal control**

I. Introduction

The unprecedented travel restrictions due to the COVID-19 pandemic started in the last month of 2019 has highly affected the aviation industry. At the time of writing this article, its adverse effect on this industry is expected to continue at least to the end of 2021. This harsh situation not only decreased the total number of passengers, but also creates the expectation of low payload ratio flights due to physical distancing which is a highly recommended requirement to prevent disease transmission especially in long haul flights. In recent years airliners' formation flight is recognized as a solution to reduce fuel consumption and its subsequent pollution. This reduction in fuel consumption occurs for trailing aircrafts owing to their exposure to the leading aircrafts' wingtip vortices. Fuel consumption accounts for about 20 to 30 percent of an airline's costs. Cut a small percentage of fuel costs leads to saving million dollars in large airlines which could be helpful especially in this pandemic period. Moreover, aircrafts participating in a formation flight follow the same route in the formation phase, and since they make up a single point on the radar screen, the navigation controller commands for them are expected to be the same. Hence, even though formation flights can increase controllers' workload in the case of aircrafts' position rotation, it can totally reduce controllers' workload by decreasing the number of points seeking navigation. If we consider the Covid-19 pandemic as temporary, according to the International Air Transport Association (IATA), the trend in aviation before this pandemic shows that the number of air passengers will almost double by 2037 and will reach 8.2 billion passengers per year [1]. This increase in demand will increase supply and, as a result, increase the workload of the controllers. Therefore, reducing the workload of the controllers will be increasingly important, and if the formation flight is implemented, the concerns in

this area will be reduced. On the other hand, if such a pandemic continues and physical distancing between passengers remains as a requirement in aviation industry, airlines will have to reduce their seats per aircraft which could decrease aircrafts' payload ratio. In this case, cutting fuel consumption by formation flight could be considered a solution. However, besides the significant benefits of formation flight, two major problems remain; the first is how the benefits of the arranged flight are distributed among the participants, and the second is how Traffic Collision Avoidance System (TCAS) can be involved in the formation flight. The air collision avoidance system can be considered as an obstacle to the formation flight and its benefits. This system determines how close the planes could get to each other. As a result, it restricts the extent of induced drag reduction in trailing aircrafts. It could also affect aircrafts maneuvers in advance rendezvous point and after separation point. However, despite the determining role of TCAS, the behavior of this system during formation phase is an issue that has not been addressed in previous research on formation flight path optimization. Two major objectives followed in this work are: first, examining the role of the TCAS in a formation flight, shows how TCAS will react to formation flight, and to what extent can affect formation flight benefits. Second, how we can control the formation in order to benefit both aircrafts.

This paper is divided into six sections. The first section is the introduction, The second Section gives a brief overview of existing literature. In the third section the main problem is defined. The forth section discusses the interference between TCAS and aerodynamic efficiency. In the fifth section a case study is presented and in the last section a conclusion is provided.

II. Literature Review

The effect of formation flight on decreasing induced drag and reducing fuel consumption is a well-known concept. However, in close formation flight such as formations common to fighter aircrafts, the risk of a collision between two airliners within the formation phase is too high. To address this problem, in 2011, Ning [2] first introduced the idea of extended formation flight for airliners, in which the distances between aircrafts in multiple formation flights are between 10 and 40 times and in double formation flights is up to about 75 times the wingspan of each airplane. Migratory birds are often seen flying together in long, arranged directions when migrating. At first, though, the reason was not discovered; by observing the formation flight of birds, aerodynamic experts hypothesized the correct reduction of the birds' energy consumption in the arranged flight. In 1914, Wieselsberger for the first time succeeded in explaining the mechanism of energy consumption using the idea put forward by Ludwig Prandtel [3]. Then in 1970, Lisman and Schulenberger showed that the optimal arrangement of three birds in a formation flight could increase the range of birds by up to 25 percent, while the formation flight between 25 birds could increase the range of birds by up to 70 percent compared to single flight mode [2] [4]. Experimental studies in this field confirmed the research. In 2001, an automated flight arrangement project was conducted by NASA to test the extent of drag reduction and the development of control commands when creating flight arrangements. The project exceeded its target by showing a 10% reduction in fuel consumption for the follower aircraft and a maximum reduction in fuel consumption of 18% in the case of two F/A-18 aircraft [5]. In recent years, the idea of formation flight has also attracted the attention of airplane manufacturer companies. In September 2014, Airbus conducted the close formation flight of five A350-900 unmanned aircrafts under VFR rules at altitudes less than 10,000 feet [6]. After the introduction of extended formation flight by Ning in 2012 [2], research shifted to further studies on extended formation flight in passenger aircrafts. Most research in this area in recent years has focused on planning and organizing network-scale formation flight. Research in this area used to be often done with the two goals of optimizing the overall route based on simplified aircraft models and finding the right options to form the formation flight from various flights. But these simple models did not have the precision needed to model effects such as aircraft latency, collision avoidance system, and aircraft rotations. In recent years, a series of pieces of research on flight arrangement have been conducted at the Sharif University of Technology, where the concept of multi-agent systems and online solutions introduced for the first time in this field [7]. In 2017, Hartjes et al. [8] investigated the multi-phase optimal control problem of two aircrafts formation flight, with the aim of minimizing fuel consumption and flight time. To model the governing equations, the authors considered the plane as a point mass in three dimensions. They also assumed a flat, non-rotating ground as an inertial frame, and the presence of wind is excluded. Their case study is two long-term flights assuming that the origin and destination airports of the flight routes are close to each other, and the flights are such that the aircrafts can form a formation flight with a slight deviation. It is also assumed that the routes of the planes are based on minimizing the cost on a great circle. In this reference, the leading and the trailing aircrafts are known from the beginning and considered as the inputs of the system. Since the formation flight in airplanes does not take place in the take-off phase from the airport of origin or approach to the destination airport, the take-off and landing phases of the aircraft have been ignored

and the starting and ending points of the route in this research are considered the entry and exit points of the Terminal Movement Area (TMA) at the altitude of 10,000 feet. Hartjes and his colleagues in the first part of this study, assumed the rendezvous exact time as an input, based on which the optimal departure times is determined. In the second part, the rendezvous time is not determined, and the effect of delay in the departure time of each aircraft is investigated. It is noteworthy that in this piece of research, the dynamics of the trailing aircraft is estimated using the leading one, and the aircrafts are considered as a single point in trajectory optimization problem. In 2018, Liu et al. [9] studied the effect of position rotation on formation flight performance. This study depicts position rotation during formation flight can reduce fuel consumption by about 11% in comparison to the case with no position rotation. In 2020, Malaek et al, [10] proposed a new three-dimensional resolution algorithm that by relaxing the TCAS thresholds paves the way for more efficient formation flights.

To conclude, previous works have only focused on the optimal choices of aircrafts or trajectory optimization to form beneficial formation flights, and failed to address the interference of TCAS and autopilot commands. This is while, not solving this interference, the formation flight for airliners is practically impossible. Currently,in airliners formation flight experiments TACS is inactivated which is unacceptable from the regulations point of view. Even by improving the current TCAS logic and deactivating it relative to the other existing aircrafts in the formation flight, TCAS would not be applicable anymore, since the collision risk among the aircrafts participating in a formation flight is even higher than the formation participant aircrafts and other intruders.

In this paper, the effect of TCAS on extended formation flight between two airliners is studied. For this purpose, in spite of other studies in this area, point mass dynamic equation is considered for both leading and trailing aircraft through the whole flight phases. To implement these dynamic equations, an aerodynamic model is proposed to estimate the trailing aircraft's induced drag reduction based on relative position to the other formation participant. This model provides the possibility to consider position rotations in predefined points of the formation trajectory and calculating position rotation effects on each aircraft's benefit.

III. Problem Definition

The trajectory optimization problem of the two aircrafts participating in a formation flight, can be categorized as an optimal control problem which should be stated in the form of an optimization problem to be solved by numerical methods. In order to provide this goal, in this study the Legendre-Gause orthogonal collocation method, is used.

A. Equations of motion

Following the same way as [11] to model the governing equations, the aircraft can be considered as a point mass in three dimensions and a flat and non-rotating ground as an inertial frame. The obtained three-dimensional equations of motion are as follows,

$$\dot{x} = V\cos\psi\cos\gamma \tag{1}$$

$$\dot{y} = V\sin\psi\sin\gamma \tag{2}$$

$$\dot{h} = -V\sin\gamma \tag{3}$$

$$\dot{V} = \frac{T-D}{m} + g\sin\gamma \tag{4}$$

$$\dot{\psi} = -\frac{L\sin\phi}{mV\cos\gamma} \tag{5}$$

$$\dot{\gamma} = -\frac{L\cos\phi}{mV} + \frac{g\cos\gamma}{V} \tag{6}$$

$$\dot{m}_f = -FC \tag{7}$$

where x and y indicate the aircraft's position in the two-dimensional inertial frame coordinate, V is the aircraft's true airspeed, which equals to aircraft's velocity in inertial frame in the case of no wind.ψ, and γ depict the aircraft's heading angle and path angle respectively. T, D, and L are the thrust, drag and lift forces respectively. m indicates the mass of the aircraft. ϕ is the aircraft's pitch angle, m_f is the aircraft's remaining fuel mass, and FC is the fuel consumption rate.

Since the focus of this research is on how to form, continue and complete a formation flight in the aircraft's cruise phase, to reduce the computational cost, the aircrafts' altitude is assumed to be fixed. This means that the origin and destination airports are assumed to be at flight altitude, although this is far from the case, however, since the formation flight takes place in the cruise phase, the fuel consumption of the climb phase can be entered into the calculations as a fixed error. Therefore, the three-dimensional equations can be simplified for moving in a two-dimensional plane by considering the aircraft path angle equal to zero in the whole flight. The resulting equations of motion are as follows,

$$\dot{x} = V\cos\psi \tag{8}$$

$$\dot{y} = V\sin\psi \tag{9}$$

$$\dot{V} = \frac{T-D}{m} \tag{10}$$

$$\dot{\psi} = -\frac{L\sin\phi}{mV} \tag{11}$$

$$\dot{m}_f = -FC \tag{12}$$

The thrust force as a function of engine control setting variable η, $(0 < \eta < 1)$ can be calculated using the following equation,

$$T = (T_{max} - T_{min})\ \eta + T_{min} \tag{13}$$

in which the T_{min}, and T_{max} are the maximum and idle values of the engines' thrust.

As a result, in this study, five state variables including the aircraft's position x, and y in the two-dimensional coordinate, the aircraft's velocity V, the heading angle ψ, and the remaining fuel mass of the aircraft m_f and two control variables consist of the engine control setting variable η, and the aircraft's pitch angle ϕ are considered. The lift force in the Equation 11 is calculated using the following equation,

$$L = C_L qS = C_L \frac{1}{2}\rho V^2 S \tag{14}$$

where, C_L is the lift coefficient and assumed to be constant, q is the dynamic pressure, S is the aircraft's wing area. According to reference [2], in the case of solo flight, the ratio of induced drag to total drag can be approximated from the following equation,

$$\left(\frac{D_i}{D}\right)_{solo} = \frac{L^2}{q\pi b^2 eD} = \left(\frac{L}{D}\right)\left(\frac{W}{S}\right)\left(\frac{1}{AR}\right)\frac{1}{q\pi e} \tag{15}$$

in which, D_i is the induced drag, b is the wing span, e is the Oswald efficiency, and AR is the wing aspect ratio of the aircraft.

In a formation flight, the trailing aircraft's induced drag drops. However, the non-induced part remains the same. As a result, the following equation can be used to calculate the aircraft's drag. (in the case of leading aircraft, D_{i_F} equals to D_{isolo})

$$D_F = \ D_{i_F} + \left(\frac{D_{i_{Solo}}}{\left(\frac{D_i}{D}\right)_{solo}}\right) - D_{i_{Solo}}) \tag{16}$$

where, D_{i_F} is the induced drag, and D_F is the total drag, both in the case of formation flight. The induced drag force is calculated as follows,

$$D_i = C_{D_i} qS = C_{D_i}\frac{1}{2}\rho\ V^2 S \tag{17}$$

where, the solo flight induced drag is calculated using the following equation,

$$C_{D_i} = \frac{1}{\pi\ AR\ e\ C_L^2} \tag{18}$$

To calculate the induced drag coefficient in the case of formation flight an aerodynamic model is introduced in V-A.

B. Formation Flight Modelling

In the multiphase approach, the path of each aircraft is divided into three phases: solo flights before the rendezvous point, formation flight phase, and solo flights after the separation point. This approach is similar to the reference [8] approach to model this problem. The formation flight phase is common between the two aircraft, so a total of 5 flight phases are considered in this approach, as shown in Figure 1. The first and second phases are the solo flight phases of the first and second aircraft ending at the rendezvous point, the third phase is the formation flight phase, and the fourth and fifth phases are the solo flight phases of the first and second aircraft from the separation point to their destinations. Therefore, as it can be seen in Figure 1, four connection points can be introduced including the connection between phase 1 and phase 3, phase 2 and phase 3, phase 3 and phase 4, phase 3 and phase 5. Each connection point means synchronous final time of the first phase and initial time of the second phase. So, we can say $t_{f_1} = t_{f_2} = t_{rendezvous} = t_{0_3}$, and $t_{f_3} = t_{0_4} = t_{0_5}$. Therefore, the equality of the state variables at the points of connection between phases is the other set of constraints that should be considered in our problem.

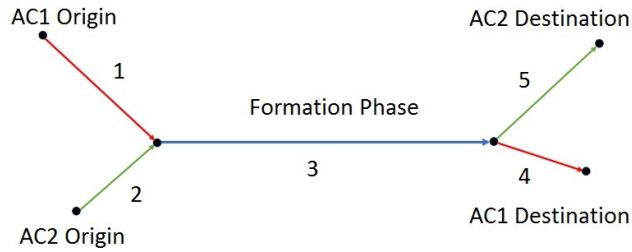

Figure 1: The schematic of the formation flight trajectory

$$\underset{t_{0_i}, t_{F_i}, x(t), u(t)}{min} \quad \sum_{i=1}^{5} w_{i_t}(t_{F_i} - t_{0_i}) + \sum_{i=1}^{5} w_{i_{FC}} \int_{t_{0_i}}^{t_{F_i}} FC_i(\tau, x(\tau), u(\tau))\, d\tau \tag{19}$$

where the first summation is the Mayer term, and the second one is the Lagrange term.F_{C_i} is the fuel consumption rate in phase number i of the formation flight. t_{0_i} and t_{F_i} are the initial time and final time of the phase number i respectively. Also, w_{i_t}, and $w_{i_{FC}}$ are sequentially the time importance factor and fuel consumption importance factors in phase i.

The fuel consumption rate in the formation phase 3 is considered to be the sum of both aircraft's fuel consumption rate. The fuel importance factor is the same in all five phases. However, the time importance factors considered to be the same in solo phases and different from the formation phase 3. As a result,

$$w_{1_t} = w_{2_t} = w_{4_t} = w_{5_t} = w_{s_t} \tag{20}$$

$$w_{3_t} = w_{f_t} \tag{21}$$

where, w_{s_t} is the Mayer term coefficient in solo phases, and w_{f_t} is the Mayer term coefficient in the formation phase. The constraints of this problem consist of, the aircraft dynamics estimation error, path bounds on states and controls, aircraft's maximum allowable Mach number and TCAS constraints as path constraints, and the equality of state variables at the linkages.

IV. Investigation of Formation Flight and TCAS Interference

In order to explore the interference between the formation flight and the collision avoidance system in this research, in the first subsection, the induced drag estimation model is explained, and at the second subsection a brief review of TCAS's logic is provided.

A. Induced Drag Estimation Model

One of the main challenges of this research was to create an induced drag estimation model and incorporate it into the path optimization process. Since the optimization process in a variety of methods requires continuity in the constraints and objective functions as well as their gradients, it is necessary that the induced drag estimation model and its gradients be as continuous as possible, without drastic changes. Because in case of discontinuity or severe changes, the solver stops the solving process before finding an acceptable answer [12].In such circumstances the use of interpolation at first glance seems appropriate but using interpolation to determine the drag induced optimization problem causes an unacceptable increase in computational cost to the extent of days will be (even while using high performance calculation systems). Therefore, in this study a simplified aerodynamic model is used.

In this model, changing in the aircraft induced drag due to its relative position to the other aircraft is estimated. This model is based on the reference [2] which was discussed in the literature review section. According to the American Federal Aviation Administration, in 95% of the flight cases, null to low turbulence conditions prevail approximately [13]. Assuming that the formation flight is performed at low turbulence values, and high performance vortex tracking, according to reference [2], the diagram of Figure 2 is used to estimate the induced drag with respect to the distance between the two aircrafts.

Figure 2 shows the changes in the leading and trailing aircrafts average induced drag ratio. In order to achieve the induced drag reduction percent in the trailing aircraft using this average value, the following equation can be used,

$$\frac{C_{D_{i_T}}}{C_{D_{i_s}}} = \frac{nC_{D_{i_t}}}{C_{D_{i_s}}} - 1 \tag{22}$$

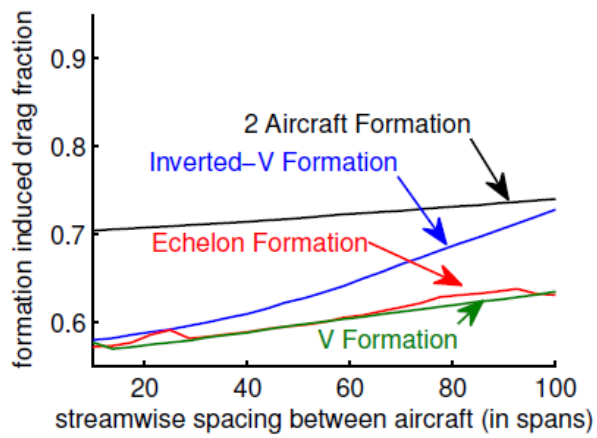

Figure 2: Induced drag reduction along the flow [2]

where, $\frac{C_{D_{i_t}}}{C_{D_{i_s}}}$ is the average induced drag ratio presented in Figure 2. n is the number of aircrafts participating in the formation flight which equals to 2 in our problem, and $\frac{C_{D_{i_T}}}{C_{D_{i_s}}}$ is the desired induced drag reduction ratio of the trailing aircraft. Since in this research it is assumed that the vortex tracking is done accurately, the exact location of the vortex is not discussed in this research, and to facilitate the calculation of the optimization algorithm, space twice the wingspan of the aircraft is intended for vortex presence. Figure 3 depicts the induced drag reduction estimation model in which the leading aircraft and the variations of induced drag to solo flight induced drag are plotted at different points regarding the leading plane. As can be seen, the induced drag reduction exists up to 75 times the wingspan along the flow direction.

Therefore, the closer the trailing aircraft is to the leading aircraft, the more it will benefit from the formation flight by further reducing the induced drag. But the collision avoidance system limits the allowable proximity.

B. Constraints due to Collision Avoidance System

According to reference [14], Traffic Advisory or Resolution Advisory alerts are displayed in the collision avoidance system only when both the range tau and the vertical tau are less than the thresholds set based on the sensitivity level. According to Figure 4, taken from reference [2], the vortex at the rear of the aircraft moves down only about 0.1 times the wingspan at a distance of 20 times the wingspan. Estimating this motion as a linear one in a short period, the vortex would moves down only 0.4 times the wingspan. Currently the longest wingspan is the A380 aircraft equals to 80 meters. 0.4 times of this length is 32 meters or 105 feet, which is less than the ZTHR altitude threshold of 850 feet considered in the reference [14]. Therefore, the vertical tau in the formation flight is always less than the threshold value,

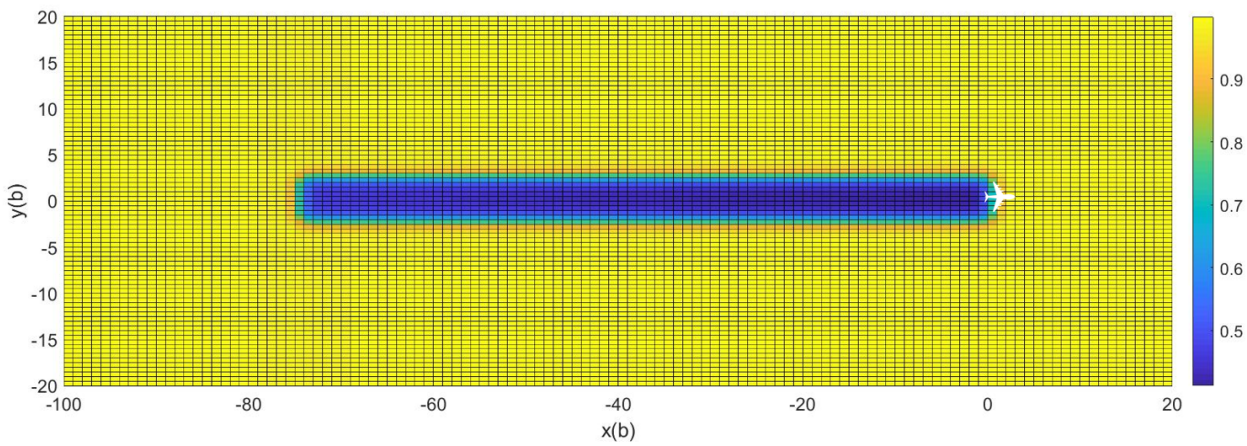

Figure 3: The induced drag reduction model

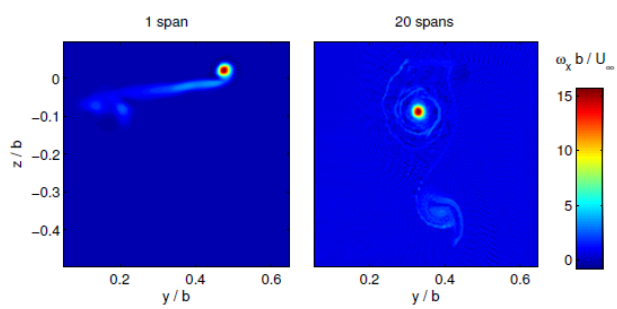

Figure 4: The vortex motion along the flow [2]

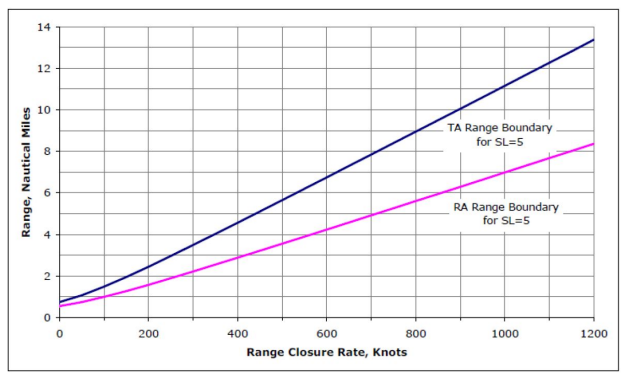

Figure 5: TA/RA Range Boundaries for 5000ft to 10000 ft altitude [14]

and the range tau would be the only factor in activating the collision avoidance system.

In this study TCAS constraints considered as path constraints in which first the aircrafts' relative velocity is calculated using the velocity values and heading angles, then by using a linear approximation similar to Figure 5 for altitudes higher than 20000 feet, the recommended minimum distance is calculated. So, the TCAS constraint does not let the aircrafts get closer than the minimum calculated threshold.

V. Case Study

In this study, the formation flight between two identical aircraft is examined and the performance of the aircraft is considered similar to that of the B777-300ER aircraft. The choice of this aircraft is due to the fact that there are currently 806 of this type of aircraft in the fleet of airlines around the world, and since the first delivery of this type of aircraft was done in 2004 and its total orders have grown in these years [15], It seems that this aircraft will continue to be one of the most operating aircraft in the world until 2037, the year mentioned in IATA forecasts. According to IATA forecasts, by 2037, the center of air traffic will shift to the east [1]. So it looks like there will be more flights between major Chinese cities and European cities in the coming years. Therefore, as it can be seen in Figure 6, two routes, Frankfurt-Shanghai and Rome-Guangzhou, were considered for this study. The cities of Shanghai and Guangzhou are currently the first and third most important economic centers in China, respectively [16]. The distance between two sources and the distance between two destinations of aircraft are close to ten percent of the route traveled by solo flights.

At the beginning of this chapter, the assumptions are described. Later, a general result obtained from the solution is examined. Afterward, each of the effective and controllable parameters in solving the problem is examined separately investigating the changes of each parameter and its effect on the final answer. Problem inputs include the initial and final conditions of the aircraft, the intended altitude for flight, the load factor

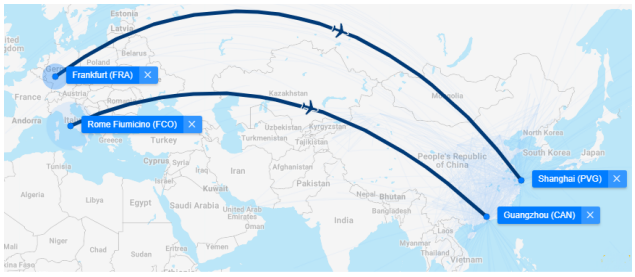

Figure 6: Case study routes

Parameter	Value
Altitude	36000 ft.= 10973 m
MTOW	351,533 kg
Fuel Capacity	145,538 kg
A1 Load Factor	0.8
A2 Load Factor	0.8
C_L	0.5
Oswald efficiency number	0.8
TSFC	2.38e-05 kg/(N.s)
Max Thrust (SL)	512,879 N
Max Mach Number	0.89
Lift to Drag Ratio	24.4

Table I: Important parameters' values

of each aircraft, the aerodynamic characteristics, and the performance specification of the aircraft including maximum take-off weight, maximum fuel tank capacity, operating empty weight, thrust specific fuel consumption, maximum thrust and the maximum operating Mach number of the aircraft. The values of the inputs mentioned in the problem are given in Table I. Aircraft 1 flies Rome-Guangzhou and aircraft 2 flies Frankfurt-Shanghai. Also, the amount of reserved fuel required for both aircraft at their destinations considered to be 5% of the tank capacity.

In this solution, the aircrafts are allowed to rotate once at a commanded point in the middle of their trajectory. The outputs of the problem include the state variables, control variables, and the time at the start point, rendezvous point, separation point, and final point. Figure 7 shows the solution obtained as the optimal aircrafts' path. O_i indicates the origin of aircraft number i and D_i indicates the destination of aircraft i in the plane.

Since the distances between the aircrafts are equal to times of the aircrafts' wingspans and are small compared to the overall scale of the coordinate display, Figures 8 to 10 show the exact position of the aircrafts relative to each other.

Figure 8 shows the position of the aircrafts relative to each other after the rendezvous point, Figure 9 shows the position of the aircrafts relative to each other at the position rotation ponit of the two aircrafts , and Figure 10 shows the position of the aircrafts during separation. Figure 8 also shows that the TCAS constraint causes the starting point of the formation flight to be approximately 200 km far from the rendezvous point.

Figure 11 to Figure 15 show how the variables of velocity, heading angle, and fuel mass of the aircraft, as well as control variables of engine adjustment and pitch angle for both aircraft in different phases change over time. The start and end points of the formation flight are shown with black hollow circles for aircraft1 and with black stars for aircraft2. Both aircrafts are assumed to start with a speed of 900 km / h.

In the diagram of the speed of the two planes in Figure 11, in the position rotation point, aircraft 2 increases its speed by about 10 kilometers per hour to overtake aircraft1 and returns to the previous speed. It is also observed that the velocity in single phases is higher than the speed in flight mode. The reason is related to how to adjust the parameter of time importance in single phases and in formation flight.

In the heading angle diagram Figure 12, as expected, the heading angles of the two aircrafts in the formation flight are approximately equal. However, they change slightly when aircraft 2 overtakes aircraft 1. Also, at the beginning of the formation flight, the heading angle adjustment can be seen which shows the effect of TCAS constraints.

In the diagram related to the changes of η control variable 13, it can be seen that initially the value of this variable in aircraft 1 is higher than the equivalent value of aircraft 2 due to being the leading plane. Then due to the position rotation of planes, it becomes vice versa. The general trend of increase in this chart is due to weight loss. Weight loss in this case according to the relation 15 reduces the ratio of induced drag to total drag. Since the effect of formation flight is only on induced drag, reducing this ratio increases the total drag with weight loss. Increasing the total drag also increases the required propulsion force and consequently increases the control variable.

In the diagram related to fuel mass in airplanes Figure 14, as expected in the formation flight phase, first the fuel consumption rate in aircraft 1 is higher due to being the leader and then due to the rotation of aircrafts, the fuel consumption rate in aircraft 1 is lower. Fuel consumption rates in single flight phases also vary depending on the speed of the aircraft.

Also, in the diagram related to the pitch angle control variable Figure 15, it is observed that the aircrafts' pitch angles become non-zero at the beginning and end points of the formation flight.

In the absence of formation flight and flying on the great circle that forms a straight line on the flat plane,

aircraft 1 needs 104,104 tons of fuel for its route, and aircraft 2 needs 107,694 tons of fuel. These amounts of fuel have reached 94,904 tons and 98,130 tons of fuel, respectively, by forming a formation flight. This result indicates an 8.86% reduction in total fuel, an 8.84% reduction in fuel consumption for aircraft 1 and an 8.88% reduction in fuel consumption for aircraft 2.

A. Sensitivity analysis

1) DMOD: Limitation of range tau in the collision avoidance system at different levels of sensitivity at zero relative velocity tends to constant values called DMOD. Assuming flying at altitudes above 20,000 feet, this constant value is 1.3 nm for traffic alerts. The optimization algorithm to get the most out of the formation flight brings the planes as close together as possible, keeping the relative speeds of the two planes close to zero at most points solved in the formation flight phase. In this case, the allowable distance set in the system is 1.3 nm. Depending on the type of aircraft, this distance is equal to different coefficients of the wingspan. On the B777-300ER, this distance is about 37 times the wingspan of the aircraft. On a medium-range aircraft such as the B737-400, this value reaches about 83 times the wingspan, and therefore, the collision avoidance system practically prevents the formation flight. To form formation flights in these cases, it is necessary to reduce the DMOD values.

If the amount of DMOD decreases to 1.2 nautical miles, the reduction in total fuel consumption will reach 9.11 percent, which is an increase of 2.6% compared to DMOD equal to 1.3. Also, if the amount of DMOD decreases to 1.1 nautical miles, the total fuel consumption reduction will reach 9.24%, which is an increase of 4.1% compared to DMOD equal to 1.3.

2) Position Rotation: In the case study, it was assumed that first the aircraft1 is the leader. If we assume aircraft2 would be the leader at the first segment of the route and then the position rotation happens, total fuel consumption is reduced by 8.16%, which is 8.1% less decrease than the the case the that aircraft 1 was the leader at first.

If the aircraft1 be the leader in the whole formation phase, the total decrease in fuel consumption would be 9.19% , which is about 3.5% higher decrease than the case performing position rotation. This difference is due to the impact of the collision avoidance system during the position rotation process. This system leads the position rotation process occupy a distance in the middle of the route and reduces the total distance traveled in formation flight mode, thus reduces the overall benefit of the formation flight.

VI. Conclusion

In this paper, trajectory optimization has been done for a dual identical aircraft formation flight considering TCAS constraints. In order to model the TCAS constraints in formation flight, a program is developed in MATLAB environment using GPOPS [17] software. This program uses an induced drag reduction estimation model for each aircraft in different relative positions to the other aircraft.

The results of this study indicate that with the presence of current collision avoidance systems, profitable dual formation flights can be formed. However, since the TCAS thresholds in different flight altitudes are independent of the aircraft type, the wingspan length plays an important role in the TCAS and the aerodynamic benefits interference. The longer the wingspan, the two aircrafts distance can be stated as a smaller factor of the wingspan, so from an aerodynamic point of view a higher induced drag reduction can be achieved. However, it seems a reliable system for tracking the leading aircraft vortex is required to better manage aircraft formation flight.

In addition, examining the effect of changes in the logic of the collision avoidance system, shows that by reducing the allowable distances by less than ten percent at zero relative velocity, Distance MODification (DMOD), in comparison to the solo flight case 2.5% more fuel can be saved.

According to the results, it can also be stated that aircraft maneuvers at the beginning and end of the formation flight phase and during the position rotation of aircrafts are affected by the collision avoidance system constraints. It includes the restrictions imposed by the collision avoidance system during position rotation which reduce the maximum achievable benefit. However, the position rotation can still be defended as a principled solution for sharing the benefits of formation flight, as well as a solution for managing the added management burden of formation between two aircrafts. Furthermore, the written program provides us the capability of building connections between the route waypoints and position rotation, in a way that the position rotations can be accomplished at any desired waypoint to share the formation flight benefit.

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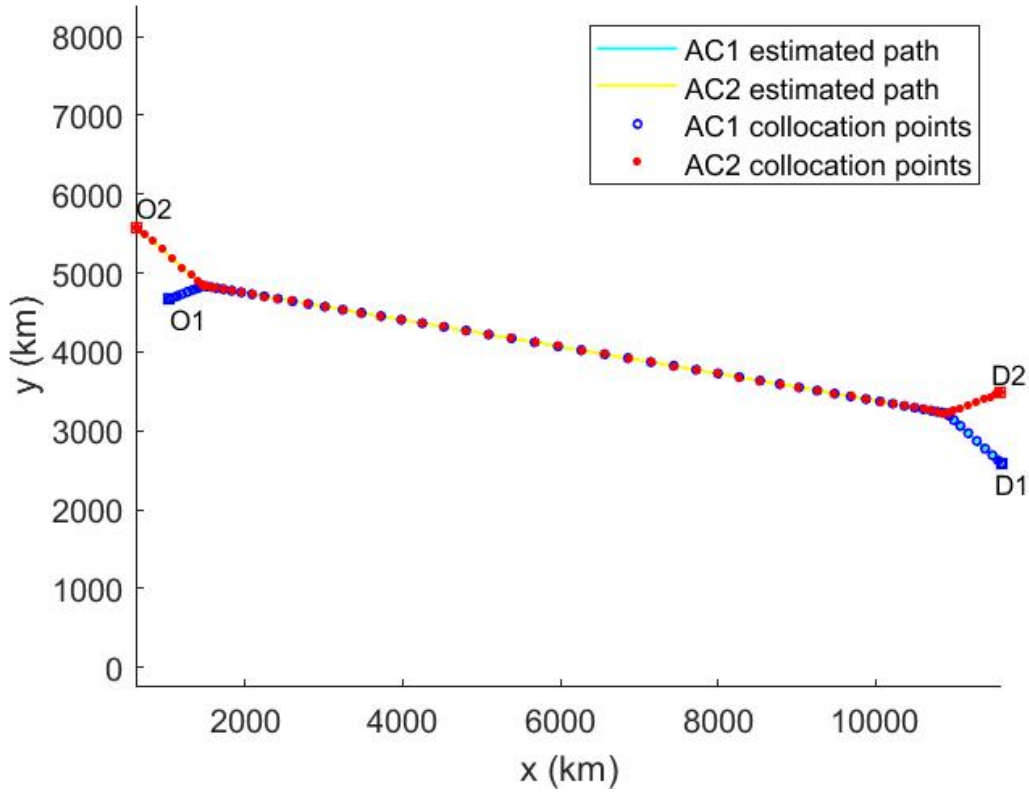

Figure 7: The obtained trajectory

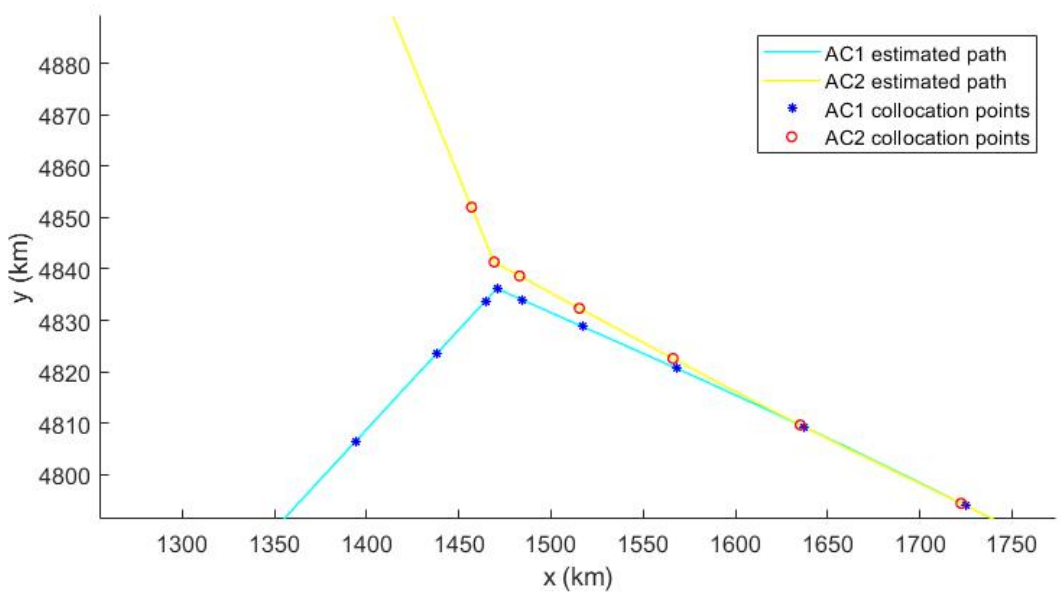

Figure 8: The aircrafts' behavior in the rendezvous point

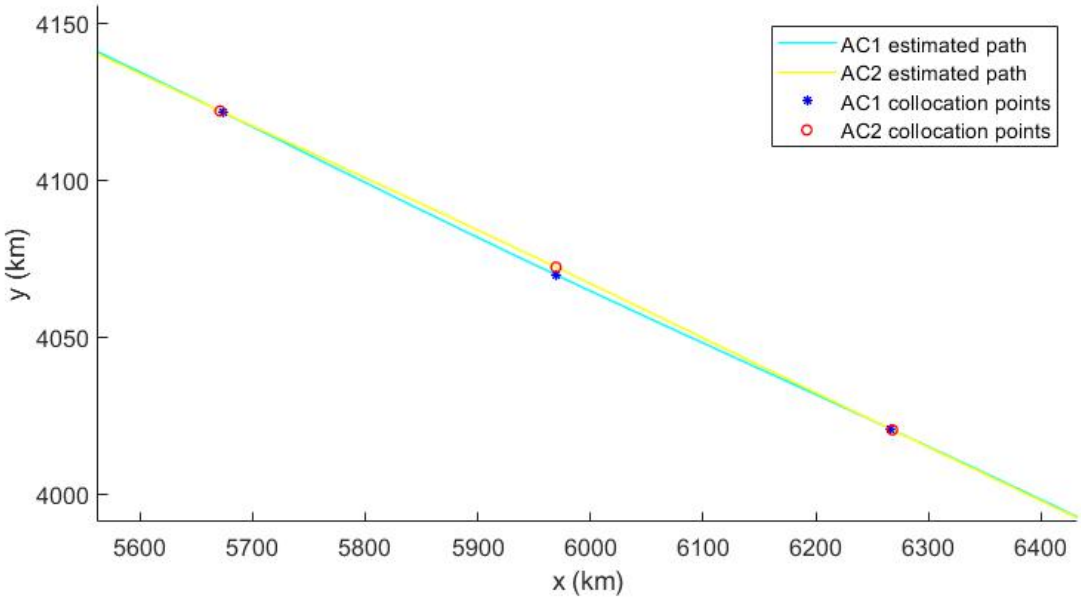

Figure 9: The aircrafts' behavior while position rotation

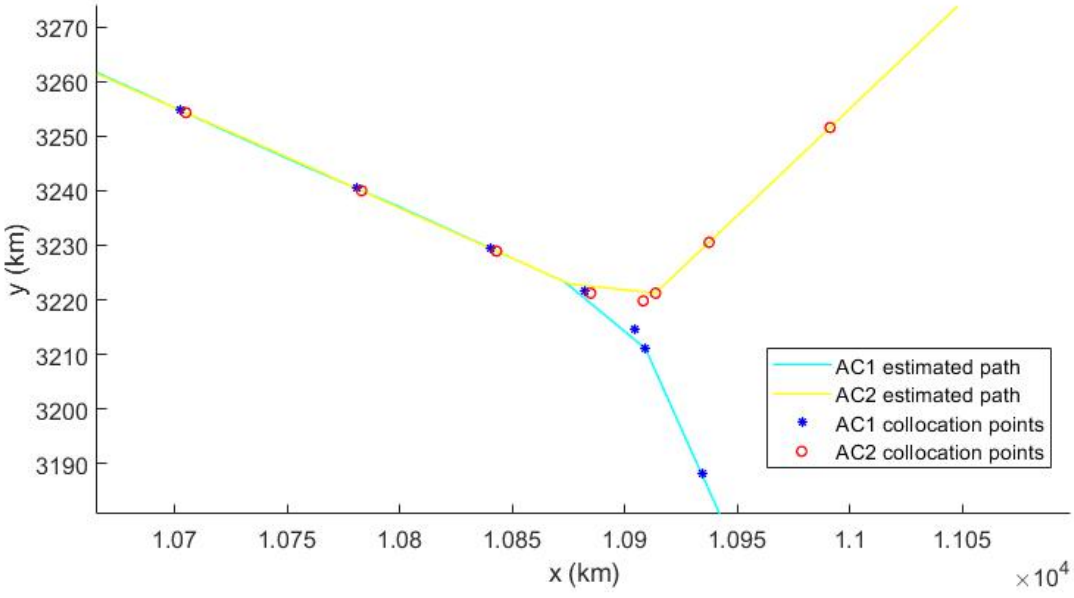

Figure 10: The aircrafts' behavior in the separation point

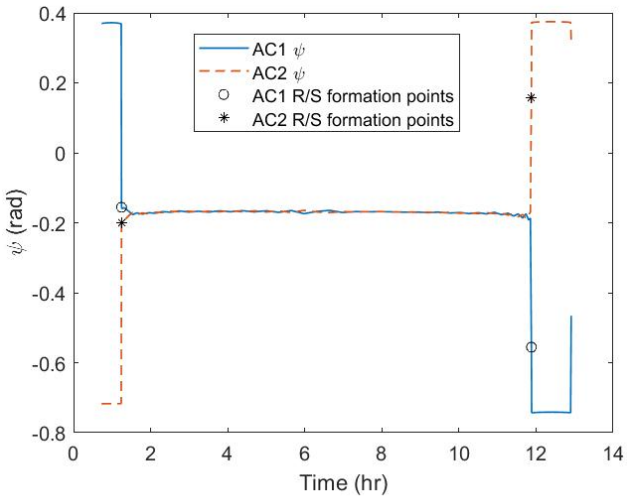

Figure 11: The change in aircrafts' heading angle in their whole flight

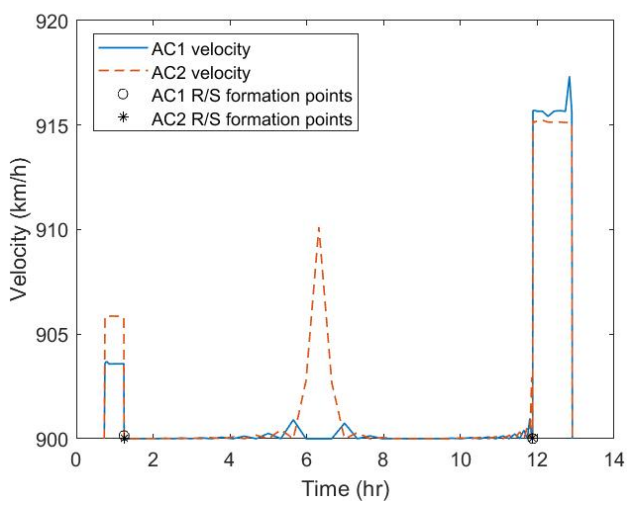

Figure 12: The change in aircrafts' velocity in their whole flight

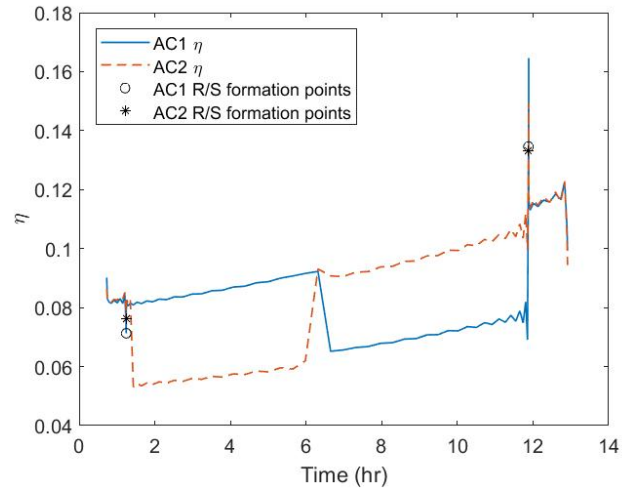

Figure 13: The change in aircrafts' η in their whole flight

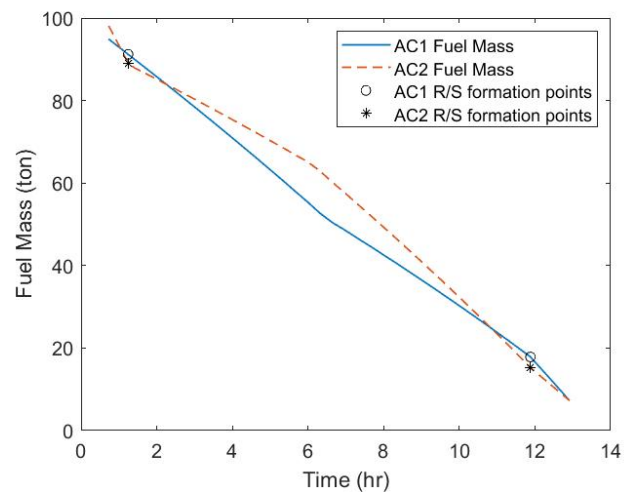

Figure 14: The change in aircrafts' fuel mass in their whole flight

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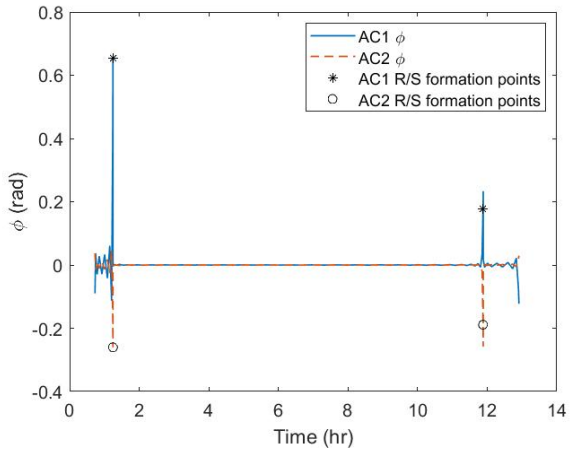

Figure 15: The change in aircrafts' pitch angle in their whole flight

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