MODEL UPDATING OF AN AIRCRAFT ENGINE SUPPORT FRAME

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The applicability of a recently introduced model updating methodology was assessed on an aircraft engine support frame. A greater than three-fold increase in the number of design variables and a greater than two-fold increase in the number of eigenvalues were considered to examine the impact of a larger design space and more computationally demanding fitness function. The experimental data was refined to eliminate four erroneous degrees of freedom. A multi-objective optimization technique was used to iteratively manipulate the computational mass and stiffness elements of the computational model towards generating a psuedo-orthogonality of identity and a minimized natural frequency discrepancy with the refined experimental data. A Pareto front of 23 optimal solutions was generated. The psuedo-orthogonality increased from > 0.716*, to* > 0.811*, to* > 0.9024 *at each stage of the methodology. This case study shows the applicability of this model updating methodology to aerospace applications investigating an increased number of DOFs and modes of interest.*

Nomenclature

ϵ_1	Function representing the normalized difference in experimental and computational natural frequencies
ϵ_2	Function representing the normalized difference in experimental and computational mode shape vectors
ω_A	Computational natural frequency [Hz]
ω_X	Experimental natural frequency [Hz]
CMP	Correlated mode pair
$CORTHOG$	Coordinate orthogonality check
DOF	Degree of freedom
EMA	Experimental modal analysis
FEA	Finite element analysis
i	Mode index
m	Number of modes in the system
$NSGA-II$	Non-dominated sorting genetic algorithm II
POC	Pseudo-orthogonality check
U	Design variables

1 Introduction

The recent surge in the application of topology optimization (TO) within the aerospace industry has increased the importance of validated computational models of aircraft components [1, 2, 3, 4, 5, 6]. Model updating is the process of updating a finite element (FE) model to better represent experimental modal analysis (EMA) results. Inherent to this methodology is the assumption that the EMA results are a perfect representation of the physical system. As a result of this limitation, the FE model often gets updated to represent an imperfect representation of the physical model. Additionally, much of literature in the model updating field has yet to be able to prove correlation between the updated FE model and the experimental data. Recent work by Warwick et al. [7] has introduced an improved model updating technique to address both limitations.

The methodology introduced by Warwick et al. [7] is the first to introduce the coordinate orthogonality check

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(CORTHOG) to iteratively remove poor measurement degrees of freedom (DOFs) from the system. The authors have demonstrated that this methodology is able to substantially improve the experimental data set prior to model updating, generating the most accurate representation of the physical model. Once the experimental data refinement is complete, a multi-objective optimization technique is used to simultaneously reduce the discrepancy between computational and experimental natural frequencies and mode shape vectors. Proof of correlation between experimental and computational mode shape vectors in ensured by use of the pseudo-orthogonality check (POC). A Pareto front of optimal solutions is generated in a single run of the algorithm, bypassing the need for weight functions. This allows for alternate solutions with differing preference in the accuracy natural frequencies or mode shape vectors to be considered. The generation of Pareto front in a single run increases the efficiency of the algorithm, and increases the accessibility of this methodology for non-experts in field of optimization by bypassing the use of weight functions.

The methodology introduced by Warwick et al. [7] will be used to investigate the feasibility and applicability of performing model updating on an aircraft engine support frame. The case study will use the computational and experimental data from Refs. [8, 9]. The data will be updated to assess the applicability of this methodology to an aerospace application considering an increased number of DOFs and modes. The following section outlines the methodology introduced by Warwick et al. [7].

2 Methodology

The methodology introduced by Warwick et al. [7] consists of two main stages. First, the CORTHOG is used to remove the inaccurate DOFs from the experimental data set until the sum of the diagonal pseudo-orthogonality terms is maximized. Human error is inherent to experimental data collection, specifically double hits and inconsistent impact location/angle. This methodology minimizes experimental error to provide the most accurate representation of the physical system.

Second, the revised experimental data set is used as the baseline to update the computational model towards. Multi-objective optimization using the non-dominated sorting genetic algorithm (NSGA-II) is performed to iteratively modify the reduced stiffness and mass elements of the system to converge towards minimization of two objective functions. The two objective functions were chosen to minimize the difference in natural frequencies between the two data sets, and the difference between the POC between the two sets of mode shape vectors and unity. The use of POC as an objective function provides proof that the computational and experimental mode shape vectors are orthogonal with respect to the mass matrix, a concept that is derived from first principles. The two objective functions (ϵ_1 and ϵ_2) are given in Eq. (1).

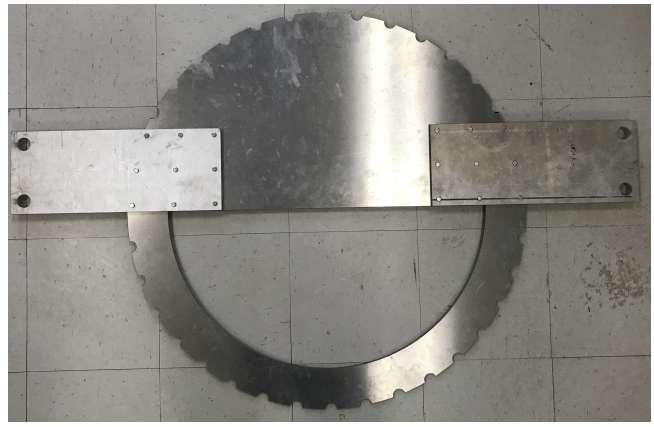

Fig. 1: Aircraft engine support frame.

$$\epsilon_1 = \sum_i^m \frac{|\omega_{X_i} - \omega_{A_i}|}{\omega_{X_i}}$$
$$\epsilon_2 = \sum_i^m |1 - POC_{ii}| \tag{1}$$

where *m* is the number of modes in the data set, ω_X are the experimental natural frequencies and ω_A are the computational natural frequencies.

An upper bound was placed on the POC to ensure physical reality of the solution. This constraint is given in Eq. (2).

$$POC_{ii}(U) \leq 1 \tag{2}$$

3 Results

3.1 Modal Analysis of an Aircraft Engine Support Frame

The experimental modal analysis of an aircraft engine support frame was collected by Chamberlain and Mechefske [8] and the computational modal analysis and model validation of the frame was performed by Warwick et al. [9]. For information regarding the methodology, data collection and analysis refer to Refs. [8, 9]. The results are summarized below. The aircraft engine support frame is shown in Fig. 1.

Table 1 displays the natural frequency discrepancy of the correlated mode pairs (CMPs) between the computational natural frequencies (ω_A) and the experimental natural frequencies (ω_X) of the aircraft engine support frame. Correlated mode pairs 3 and 4 are of primary concern as the natural frequency discrepancy was $> 10\%$.

The POC between the two data sets is given in Fig. 2. The initial results generated eight POC_{ii} terms > 0.9. The

Table 1: Natural frequency comparison for the 12 CMPs identified by Warwick et al. [9].

CMP	ω_X [Hz]	ω_A [Hz]	Discrepancy [Hz]	% Difference
1	3.21	3.23	0.02	0.62%
2	8.13	8.20	0.07	0.86%
3	10.6	11.7	1.1	10.4%
4	13.9	6.60	7.3	52.5%
5	16.3	15.7	0.6	3.7%
6	24.5	22.9	1.6	6.5%
7	29.9	28.5	1.4	4.7%
8	39.1	41.8	2.7	6.9%
9	43.1	43.6	0.5	1.2%
10	52.4	48.2	4.2	8.0%
11	59.1	63.9	4.8	8.1%
12	76.3	79.3	3.0	3.9%

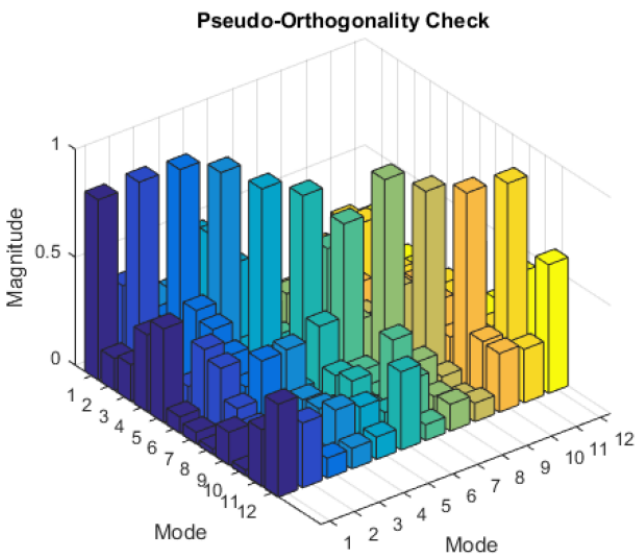

Fig. 2: POC for the aircraft engine support frame.

average POC_{ii} value was 0.897, and the minimum value was 0.716.

3.2 CORTHOG Results

The CORTHOG was calculated for the two initial data sets. Degree of freedom 50 generated the largest CORTHOG value and was eliminated from the system. This process was iterated until the POC was optimized. Optimization occurred with the removal of four DOFs from the system as the sum of all twelve POC_{ii} values was maximized. The POC_{ii} values at each iteration are given in Tab. 2.

After removal of the four erroneous DOFs, the final reduced system contained 58 DOFs. The CORTHOG reduction process increased the number of $POC_{ii} > 0.9$ from eight to nine, the average POC_{ii} value from 0.897 to 0.931, and the minimum POC value from 0.716 to 0.811.

Table 2: POC_{ii} values at each iteration of the CORTHOG DOF reduction analysis.

DOF	Original Data [9]	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5	Iteration 6
1	0.796	0.798	0.799	0.800	0.811	0.830	0.890
2	0.911	0.900	0.904	0.917	0.894	0.876	0.882
3	0.942	0.992	0.990	0.983	0.981	0.980	0.961
4	0.938	0.983	0.985	0.996	0.996	0.994	0.999
5	0.985	0.985	0.985	0.982	0.990	0.995	0.992
6	0.896	0.871	0.864	0.892	0.888	0.898	0.908
7	0.790	0.892	0.923	0.928	0.939	0.952	0.956
8	0.991	0.987	0.977	0.968	0.952	0.923	0.883
9	0.917	0.919	0.919	0.919	0.922	0.922	0.922
10	0.916	0.858	0.883	0.891	0.907	0.920	0.925
11	0.971	0.988	0.985	0.973	0.970	0.959	0.856
12	0.716	0.916	0.916	0.916	0.916	0.916	0.916
sum	10.769	11.089	11.130	11.165	11.166	11.165	11.090

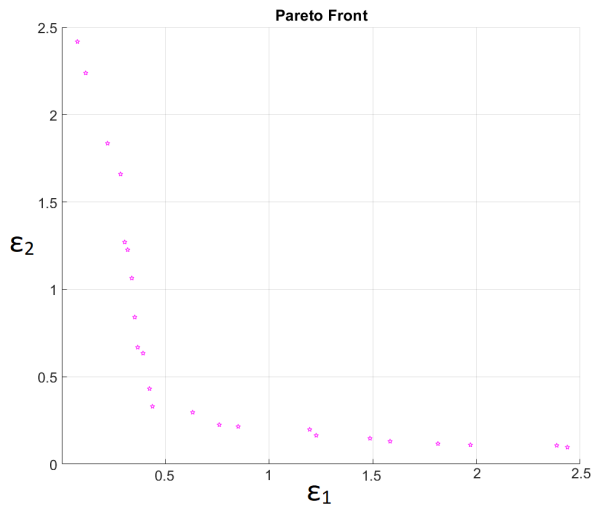

Fig. 3: Pareto front of optimal solutions.

3.3 Optimization Results

The CORTHOG reduced system was used as the initial system for the multi-objective optimization. The mass and stiffness matrices of the system were 58×58 resulting in 3364 elements in each matrix and 6728 design variables for optimization. The NSGA-II algorithm was used for multi-objective optimization. A Pareto front of optimal solutions was generated and is shown in Fig. 3. Twenty-three optimal solutions were generated. The sum of the natural frequency discrepancy for all twelve modes ranged from 8.8% to 246.2%, and the sum of the POC discrepancy ranged from 0.092 to 2.412 for the 23 solutions. Therefore, the average natural frequency discrepancy ranged from 0.73% to 20.2%, and the average POC discrepancy ranged from 0.0077 to 0.201.

The solution at the knee point of the Pareto front was examined in further detail. The natural frequency discrepancy for each mode of the knee point solution is given in Tab. 3. A significant improvement in the natural frequency correlation was noted when compared to Tab. 1. Mode 4 underwent the greatest improvement, from a 52.5% difference to a 12.2% difference.

Table 3: Natural frequency results of the knee point optimized solution.

CMP	ω_X [Hz]	ω_C [Hz]	Discrepancy [Hz]	% Difference
1	3.21	3.23	0.02	0.62%
2	8.13	8.20	0.07	0.86%
3	10.6	11.2	0.6	5.7%
4	13.9	12.2	1.7	12.2%
5	16.3	15.4	0.9	5.5%
6	24.5	22.6	1.9	7.8%
7	29.9	28.1	1.8	6.0%
8	39.1	40.4	1.3	3.3%
9	43.1	43.5	0.4	0.9%
10	52.4	51.2	1.2	2.3%
11	59.1	60.6	1.5	2.5%
12	76.3	77.7	1.4	1.8%

The POC of the knee point solution is shown in Fig. 4. The POC of the optimized solution generated a significant improvement of the original POC shown in Fig. 1, as all twelve modes of the optimized solution are > 0.9 compared to eight out of twelve modes being > 0.9 in the original POC. The minimum POC value also increased from 0.716 to 0.9024.

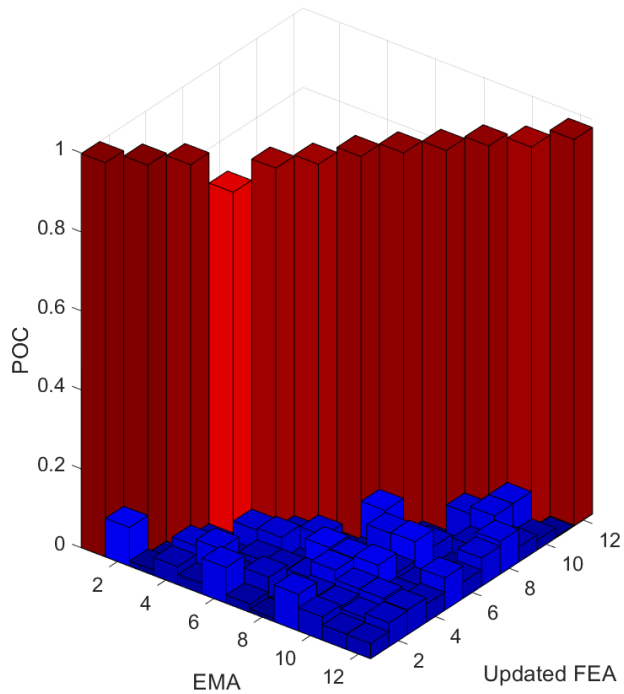

Fig. 4: Pseudo-orthogonality of the knee point optimized solution.

4 Discussion

The results of this case study imply the feasibility of the methodology introduced by Warwick et al. [7] to a variety of applications. These results show that this methodology is capable of significant improvements to a FE model that account for both experimental and computation error. This case study considered over three times more design variables than the original study and was able to generate 23 feasible solutions along the Pareto front.

The computational run-time for the multi-objective model updating was 213 minutes, an increase from the original 124 minute run-time observed by Warwick et al. [7]. This was expected due to the $> 3\times$ increase in the number of design variables, and an increase in the number of eigenvalues to from five to twelve.

The first stage of the methodology was able to remove four poor measurement DOFs. The location of two of the removed DOFs were in proximity to the contact location between the carabiners and the engine support frame during experimental testing. The experimental set-up inadvertently constrained the engine support beams that were perfectly free-free in the computational model. This caused a 52.2% natural frequency discrepancy of mode four, as mode four contained primary displacements in the engine support beams. The removal of the two DOFs in proximity to the carabiner contact location resulted in increased correlation of mode four. The other two removed DOFs were located randomly throughout the engine support frame, one on the lower ring and one on the upper plate. Neither of these two DOFs were near bolt locations, indicating that they were poor measurement DOFs due to human error, i.e. double hits, inconsistent impact location/angle, etc.

The two step methodology, to first minimize experimental error and then to minimize computational error through multi-objective optimization, was shown to be beneficial at increasing the correlation at each step. The minimum POC value of the original data set was 0.716. Upon performing the CORTHOG reduction technique, the minimum POC value was 0.811. Finally, the minimum POC value of the knee point optimized solution was 0.9024, highlighting the necessity of performing both steps during the model updating process. The multi-objective optimization process was able to refine the mass and stiffness elements of the reduced computational model, further improving the correlation beyond removal of poor measurement DOFs alone.

5 Conclusion

The methodology introduced by Warwick et al. [7] was implemented on an aircraft engine support frame. Over 3× the number of design variables, and over double the amount of modes were analyzed to assess the feasibility of this methodology with a larger design space and a more computationally demanding fitness function. The CORTHOG was able to remove four erroneous degrees of freedom, improving the POC results to be >0.811. Multi-objective optimization generated a Pareto front of twenty-three optimal solutions. The knee point solution increased the POC to >0.902 and decreased the average natural frequency discrepancy to ±1.07 Hz, a significant improvement upon the >0.716 POC and the ±2.27 Hz frequency discrepancy of the original data. The results of this case study indicate the feasibility and performance of this methodology considering an increased design space and more computationally demanding fitness function. This methodology has proved correlation between the experimental and computational data, generating a finite element model that gave the most accurate representation of the physical model. Future work can be performed to examine all applications where a significant discrepancy between a computational model and experimental data exist.

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