Design and Characterization of Micro-Structured Tensegrity Lattice Materials as a New Candidate for Skin Panels of Morphing Wings

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In an effort to increase aerodynamic efficiency of aircraft, the aerospace industry is working on developing concepts of morphing wings that can adapt flexibly to varying flight and environmental conditions. Design of skin panels particularly for morphing wings is among the main challenges that need to be addressed. It requires characteristics of both compliance and strength to endure flexible shape changing while maintaining load bearing capabilities. To this end, nonconventional micro-architectures lattice materials serve as a promising candidate. In this study, we introduce tensegrity lattice materials as a third class of periodic cellular solids and we look into the potential of their utilization in the design of morphing wings. This calls for the properties of different topologies to be examined while taking into consideration the geometrical nonlinearities exhibited. In order to understand the mechanical response of the different topologies, the comprehensive stiffness, which accounts for both the material and geometrical stiffness, of the tensegrity lattice material needs to be developed.

I. Introduction

The wing of an airplane has a significant influence on its performance in flight. The status quo involves the development of a single wing design that is most suitable to a range of flight conditions. In order to morph the shape of the wing to match the flight requirement at a certain point in the flight envelope, mechanisms like flaps and slats are deployed. However, these control surfaces cause discontinuities on the wing's surface therefore reducing aerodynamic efficiencies. On the other hand, looking to nature, we find that birds are able to morph their wings swiftly and with such elegance allowing them to control drag and lift as needed [1]. Consequently, there has been an increasing interest in the aerospace industry to transcend morphing achieved via conventional hinged mechanical surfaces to compliant adaptive wing structures. The development of a morphing wing is aimed at improving the structural and aerodynamic performance of the existing technology. It achieves a defined target by allowing small changes to the wing's geometry while sustaining external and internal stresses. To gain a deeper understanding of what goes into developing this technology, it is defined by its five components: the structural skeleton, skin, the actuators, control and sensor systems. Literature reveals that the skin, actuation and sensor systems pose the main challenges [2].

Examining the skin's function, it should provide a smooth surface to not disrupt the airflow. However, for a morphing compliant wing specifically, the skin is also required to recover from large deformations while maintaining its strength. In other words, it should be able to demonstrate high compliance in the direction of actuation while maintaining high strength and stiffness in the transverse loading direction [3]. Conventional materials like fibre glass or carbon fibre can meet one of the requirements but at the cost of others; for instance, they could offer stiffness but at the cost of compliance. Therefore, researchers are looking into nonconventional architectured materials.

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II. Lattice Materials as a Candidate

A promising candidate for morphing wings is micro-structured lattice materials [4]. These types of materials consist of a network of struts and offer the advantage of tunability to meet desired objectives. For instance, they can be manipulated to provide greater strength in areas subjected to higher loads. This is due to the governing characteristics that form their properties as shown in Figure 1. Lattice materials are governed by their constituent monolithic material whose mechanical, thermal and electrical properties influence the overall properties of the lattice, relative density which refers to the ratio of the density of the lattice to that of the constituent material and by their shape and topology which refers to the connectivity of its members. The classification of lattice materials relies on their topology.

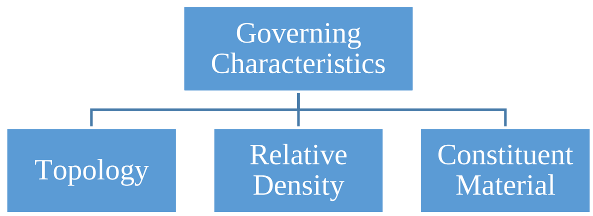


Figure 1 Governing characteristics of lattice materials

A. Stretching vs. Bending-dominated Lattice Materials and Tensegrity as a Third Class

Depending on the nodal connectivity, Z, lattice materials can be classified as either stretching or bending dominated. Lattices with high connectivity, like the cuboct elementary cell, shown in Figure 2(a), transmit loads axially and are classified as stretching dominated lattices. Structural analysis of these lattices shows that their stiffness and strength scale linearly with relative density [5], i.e.

$$\frac{\bar{E}}{E_s} \propto \frac{\bar{\rho}}{\rho_s} \tag{1}$$

$$\frac{\bar{\sigma}}{\sigma_{y,s}} \propto \frac{\bar{\rho}}{\rho_s} \tag{2}$$

where $\bar{E}$, $\bar{\rho}$ and $\bar{\sigma}$ are the relative elastic modulus, relative density and relative strength of the lattice, respectively, and E_s, ρ_s and $\sigma_{y,s}$ are the elastic modulus, density and yield strength of the constituent material, respectively. On the other hand, bending-dominated lattices are characterized by low connectivity like the regular cubic elementary cell in Figure 2(b) and transmit loads through bending. Their stiffness and strength are governed by a higher order proportionality with relative density.

$$\frac{\bar{E}}{E_s} \propto \left(\frac{\bar{\rho}}{\rho_s}\right)^2 \tag{3}$$

$$\frac{\bar{\sigma}}{\sigma_{y,s}} \propto \left(\frac{\bar{\rho}}{\rho_s}\right)^{3/2} \tag{4}$$

In comparison, stretching-dominated lattice materials are substantially more mechanically efficient than their bending-dominated counterpart. For a relative density of 0.01, a stretching dominated lattice material is a hundred times stiffer and ten times stronger than its bending dominated counterpart. It was proven that for an infinite framework to be stretching dominated, the necessary condition is Z=4 in 2D and Z=6 in 3D, whereas the sufficient nodal connectivity is Z=6 and Z=12 in 2D and 3D, respectively [6].

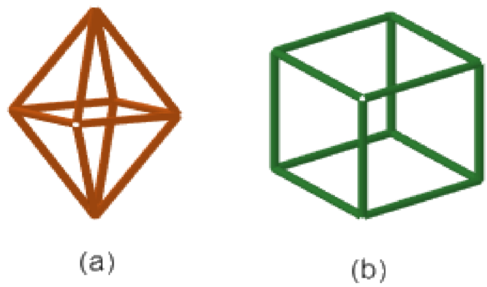


Figure 2 (a) Cuboct elementary cell (b) regular cubic elementary cell

We include in our classification, a third class of lattice material, namely, tensegrity lattice material. The concept of tensegrity has attracted the attention of researchers and engineers due to their high strength-to-weight ratios [7]. These types of lattices are based on the concept of pre-stressing. The level of pre-stress in its members serves as an additional governing characteristic where an increase in this level leads to an increase in stiffness [8].

III. Determinacy Analysis of Frames

A. Maxwell's Stability Criterion

In order to make the distinction between the aforementioned three classes, we look into the determinacy analysis of pin-jointed frameworks. Maxwell's stability criterion clarifies the minimum number of bars required for a frame to be kinematically and statically determinate i.e. simply stiff, i.e., in 2D we should have

$$b = 2j - k \tag{5}$$

While in 3D, we have

$$b = 3j - k \tag{6}$$

where b is the number of bars, j is the number of joints and k is the number of kinematic constraints. If the number of bars exceeds the minimum, then the bar is over-stiff and is statically indeterminate but kinematically determinate. Simply stiff and over-stiff frames are considered to exhibit stretching-dominated behavior. If the number of bars is less than the minimum, then the frame is a mechanism, meaning that the bars rotate about the joints and collapse under loading. However, if the joints are made rigid, the frame becomes bending-dominated.

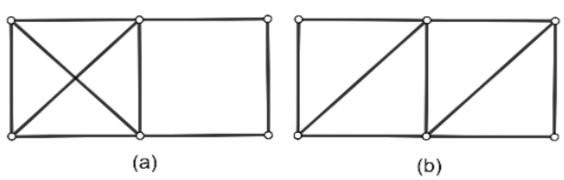


Figure 3 (a) part over-stiff and part mechanism (b) simply stiff

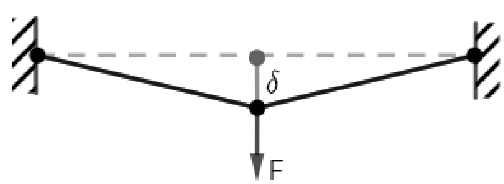


Figure 4 Pre-stressed assembly

Maxwell's rule is a necessary, but not sufficient condition for the stiffness of a frame. As can be seen in Figure 3, both 2D frameworks with $k = 3$, satisfy Maxwell's rule to be simply stiff, however the framework in (a) consists of a part that remains a mechanism. Not only that, but also the frame in Figure 4, according to the rule, is a mechanism and has zero stiffness. However, the load applied, F, is equilibrated due to the pre-stress in the elements that give rise to the system's geometrical stiffness. This stiffness, along with the material stiffness, allows for a new equilibrium configuration to be achieved after being deformed by δ from the initial one. Therefore, Calladine formed a generalization [9] to account for such structures

in 2D

$$b - 2j + k = s - m \tag{7}$$

and 3D

$$b - 3j + k = s - m \tag{8}$$

where s is the number of states of self-stress and m is the number of internal mechanisms. This criterion allows for the classification of all frames into the previously mentioned three classes, thus forming the sufficient condition for frame rigidity. Matrix methods of linear algebra are used to carry out the determinacy analysis. The number of states of self-stress and of mechanisms can be determined using the rank r of the equilibrium matrix of a structure assembly from the equations

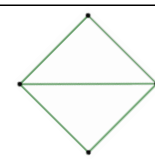
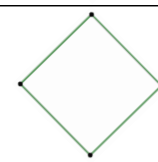
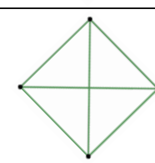
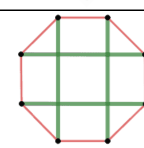
$$s = n_c - r \tag{9}$$

and

$$m = n_r - r \tag{10}$$

where n_c and n_r are the number of columns and rows of the equilibrium matrix respectively [10].

Table 1 Classification of Pin-Jointed Assemblies

<table>
<tr><th>Type of structure</th><th>States of self-stress</th><th>Internal mechanisms</th><th>Determinacy</th><th colspan="3">Topology Class</th><th>Planar Example</th></tr>
<tr><td>I.</td><td>$s = 0$</td><td>$m = 0$</td><td>Statically and kinematically determinate</td><td colspan="3">Stretching-dominated (simply stiff)</td><td></td></tr>
<tr><td>II.</td><td>$s = 0$</td><td>$m > 0$</td><td>Statically determinate and kinematically indeterminate</td><td colspan="3">Mechanism (bending-dominated if joints are made rigid)</td><td></td></tr>
<tr><td>III.</td><td>$s > 0$</td><td>$m = 0$</td><td>Statically indeterminate and kinematically determinate</td><td colspan="3">Stretching-dominated (over-stiff)</td><td></td></tr>
<tr><td rowspan="2">IV.</td><td rowspan="2">$s > 0$</td><td rowspan="2">$m > 0$</td><td rowspan="2">Statically and kinematically indeterminate</td><td rowspan="2">Stiffening effect</td><td>no</td><td>Bending-dominated (Mechanism)</td><td rowspan="2"></td></tr>
<tr><td>yes</td><td>tensegrity (pre-stressed assembly)</td></tr>
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B. Unit Cell as a Finite Structure

The equilibrium matrix $\boldsymbol{A}$ of a pin-jointed structure with b bars and j joints is a Jacobian matrix with entries of direction cosines that relates the vector of tensile forces in the bars, t, to the vector of nodal forces, f

$$\boldsymbol{A}t = f \tag{11}$$

Whereas the kinematic matrix $\boldsymbol{B}$ relates the vector of nodal displacements, d**,** to the vector of element deformations, e

$$\boldsymbol{B}d = e \tag{12}$$

By applying the principle of virtual work, it can be shown that the kinematic matrix is equivalent to the transpose of the equilibrium matrix [11].

$$\boldsymbol{B} = \boldsymbol{A}^T \tag{13}$$

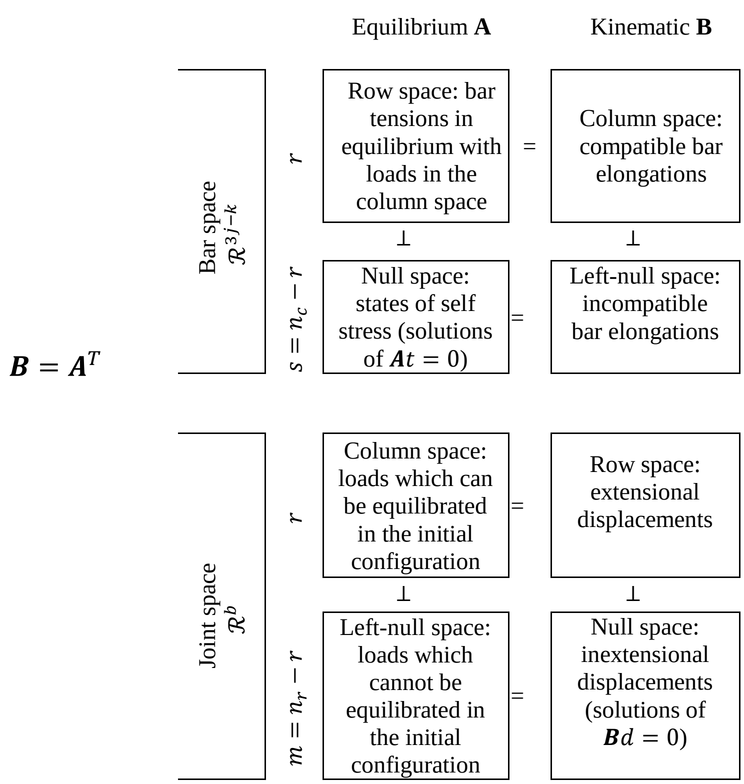

Figure 5 Duality of equilibrium and kinematic matrices where "=" shows the two subspaces that coincide, whereas "⊥" show that they're orthogonal complements. Adapted from Pellegrino [12]

This duality allows for the determination of states of self-stress in a statically indeterminate structure and mechanisms in a kinematically indeterminate structure simultaneously from the four fundamental vector subspaces of either matrix. The four subspaces of the equilibrium matrix include

- *Row space of* $\boldsymbol{A}$
  Basis of the equilibrium of the bar tensions vector $\boldsymbol{t}$ with the nodal loads vector $\boldsymbol{f}$ in the column space
  Dimension of the row space is *r*
- *Column space of* $\boldsymbol{A}$
  Basis of the equilibrium of the nodal loads vector $\boldsymbol{f}$ in the initial configuration
  Dimension of the column space is *r*
- *Null space of* $\boldsymbol{A}$
  Basis of the equilibrium of the bar tensions vector $\boldsymbol{t}$ with the nodal loads vector $\boldsymbol{f} = 0$ representing the states of self-stress, *s*, developed in the absence of external loads
  Dimension of null space is $n_c - r = s$. A dimension of 0 indicates the structure is statically determinate
- *Left null space of* $\boldsymbol{A}$
  Basis representing the inextensional nodal displacements i.e. when $\boldsymbol{e} = 0$
  Dimension of left null space is $n_r - r = m$. A dimension of 0 indicates the structure is kinematically determinate

Figure 5 clearly expresses the duality of equilibrium and kinematic matrices and their four fundamental vector subspaces. The bases of the equilibrium matrix subspaces can be found by performing a Singular Value Decomposition (SVD) in MATLAB.

$$\boldsymbol{A} = \boldsymbol{SVD}^T \tag{14}$$

where **S** is an $n_r \times n_r$ orthogonal matrix consisting of left singular vectors, **V** is an $n_r \times n_c$ matrix with *r* positive diagonal elements and **D** is a $n_c \times n_c$ orthogonal matrix consisting of right singular vectors. **S** and **D** split into further matrices: $\boldsymbol{S_r}, \boldsymbol{S_m}, \boldsymbol{D_r}$ and $\boldsymbol{D_s}$ each with its physical interpretation regarding the rigidity of the framework

- $\boldsymbol{S_r}$ communicates the loads which can be equilibrated i.e. extensional deformations
- $\boldsymbol{S_m}$ communicates the loads which cannot be equilibrated i.e. mechanisms
- $\boldsymbol{D_r}$ communicates the displacements compatible with $\boldsymbol{S_r}$
- $\boldsymbol{D_s}$ communicates the states of self-stress

The mechanisms in $\boldsymbol{S_m}$ may either be rigid-body mechanisms, *rm*, due to a lack of constraints to a foundation or internal mechanisms, *im*. Pellegrino and Calladine [13] proposed a method to distinguish between the two modes, so that $im = m - rm$ can be determined. Internal mechanisms can be finite, characterized with first-order changes of bar length, or infinitesimal, characterized with second or higher order changes of bar length. First-order infinitesimal mechanisms characterized with second order changes of bar length can be suppressed and stiffened by states of self-stress forming a pre-stressed structure. Such a structure typically responds to a load with either of two modes or a combination of both. The first mode involves a change in bar tensions resulting in a small displacement from the initial configuration, whereas the second mode involves inextensional displacement where the load is balanced by a change in structural geometry to achieve a new equilibrium configuration. The second mode gives rise to the structure's geometrical stiffness which along with the elastic material stiffness is necessary to understand the effective behavior of such assemblies. To determine if mechanisms in an assembly can be stiffened by states of self-stress to achieve a new equilibrium configuration, Pellegrino and Calladine developed the product force vector approach which serves as a necessary condition and the positive definiteness of the stress tensor quadratic form which serves as a sufficient condition. The interested reader is referred to Calladine and Pellegrino [13, 14]

C. Infinite Periodic Frameworks

The determinacy analysis of the unit cell is extended into the periodic regime to form a lattice. We make the distinction of two different types of tensegrity lattices and those are periodicity-induced and stand-alone tensegrity. The former refers to an assembly where geometrical stiffness arises due to the periodicity of the unit cell. The unit cell which may be a mechanism can develop the ability to support external loading when it is tessellated. This periodicity makes the framework statically indeterminate giving rise to states of self-stress that suppress and stiffen the modes of internal mechanisms. The latter, however, refers to a tensegrity system that is stabilized in space due to a pre-stressing of its compressive and tensile members like the planar example presented for type IV in Table 1. The elementary cell itself is both statically and kinematically indeterminate making the load bearing ability an innate characteristic.

To be able to adequately study the characteristics and performance of an infinite lattice, its continuous space must first be discretized into a discrete summation of modes giving rise to the utility of the reciprocal lattice. The bases of this lattice are developed using the direct translational bases, which are used to tessellate the unit cell to form the lattice, and the dependency relationships of the node and bar bases groups, which are used to define the unit cell and its envelope. Examining the stiffness of the periodic framework, requires the development of the reduced equilibrium and kinematic matrices. This development requires the definition of a transformation matrix that can generate periodic wave functions for the infinite lattice from finite wave functions of the primitive cell. This can be achieved by applying the Bloch theorem [15]. An in-house Matlab code developed by ElSayed [16] is used to develop the reduced equilibrium and kinematic matrices at every wave-number formulated from the irreducible first Brillouin zone of the reciprocal lattice. Examining the four fundamental subspaces of the reduced equilibrium matrix reveals the periodic states of self-stress and the periodic internal mechanisms. If the lattice is found to have internal mechanisms but not states of self-stress, it is classified as bending-dominated. If it is found to have the opposite, then it exhibits stretching-dominated behavior. If it possesses both states of self-stress and internal mechanisms, then these mechanisms are verified to be first-order infinitesimal mechanisms using the Product Force Vector (PFV) approach and the positive-definiteness of the stress tensor quadratic form. If these two conditions are met or simply the latter sufficient one, then the lattice is a tensegrity lattice otherwise it is bending-dominated. Periodic frameworks cannot lack both states of self-stress and internal mechanisms [17].

V. Effective Behavior of Periodic Lattice Material

A multiscale approach, that takes into account the physics of the microstructures, is required to obtain the macroscopic behavior. This scale transition from the micro- to the macro-level requires the employment of a homogenization technique capable of capturing the geometrical nonlinearity of the lattice material.

In his study, ElSayed employed the Cauchy-Born hypothesis to obtain the macroscopic stiffness properties of ten lattice topologies from which selection design charts shown in Figures 6, 7, 8 were developed. These charts represented data for the stretching-dominated lattice materials expressing the linear, elastic material stiffness with respect to its relative density [16]. A brief summary of the analysis conducted is provided here for the sake of completeness. The relative density of a 2D lattice material can be calculated by

$$\bar{\rho} = C_T^{\rho}\frac{H}{L} \tag{15}$$

where C_T^{ρ} is a topology-dependent density constant and H and L are geometrical parameters of the unit cell representing the width and length respectively. In accordance with the definition of the Cauchy-Born hypothesis, the microscopic nodal displacement field is expressed in terms of an assumed applied macroscopic strain field, $\bar{\epsilon}$. The Cauchy-Born kinematic boundary condition of two periodic nodes i and j, is determined as

$$\begin{bmatrix} d_{ix} \\ d_{iy} \end{bmatrix} = \begin{bmatrix} d_{jx} \\ d_{jy} \end{bmatrix} + \begin{bmatrix} (x_i - x_j) & 0 & \frac{1}{2}\,(y_i - y_j) \\ 0 & (y_i - y_j) & \frac{1}{2}\,(x_i - x_j) \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix} \tag{16}$$

Applying it to the unit cell displacement vector, d, yields

$$d = \boldsymbol{T_d}\tilde{d} + \boldsymbol{E}\bar{\epsilon} \tag{17}$$

where T_d is the nodal transformation matrix used to transform the vector of nodal displacement to its reduced form, $\tilde{d}$ is the infinite lattice reduced nodal displacement vector and $\bar{\epsilon}$ is an assumed macroscopic strain field. Substituting the unit cell nodal displacement vector into eqn (12), gives

$$\boldsymbol{B}\left(\boldsymbol{T_d}\tilde{d} + \boldsymbol{E}\bar{\epsilon}\right) = e \tag{18}$$

Applying the Bloch-wave theorem, a transformation matrix is used to obtain the reduced kinematic matrix of the infinite lattice, $\widetilde{\boldsymbol{B}}$.

$$\widetilde{\boldsymbol{B}}\tilde{d} + \widetilde{\boldsymbol{E}}\bar{\epsilon} = \tilde{e} \tag{19}$$

where $\tilde{e}$ is the reduced vector of element deformations. This formulation relates the macroscopic strain field to the periodic element deformations and nodal displacements which leads to

$$\tilde{e} = \boldsymbol{M}\bar{\epsilon} \tag{20}$$

where an empty null-space of matrix **M** indicates a stretching-dominated lattice material i.e. no macroscopic strain field is generated due to internal mechanisms.

The strain energy density of the lattice, W, derived using the element deformation vectors, is then used to obtain the homogenized stiffness tensor with entries

$$k_{iijj} = \frac{\partial^2 W}{\partial\bar{\epsilon}_{ii}\partial\bar{\epsilon}_{jj}} \tag{21}$$

where i and $j \in \{1, \dots, n\}$; n=2 for 2D lattice materials. The stretching-dominated behavior can also be verified by examining the null space of the stiffness tensor. An empty null space indicates that no macroscopic strain fields were generated under zero macroscopic stress fields. Otherwise, it would reveal the modes of macroscopic strain fields at which the framework collapses classifying it as a bending-dominated lattice material. Taking the inverse of the stiffness tensor yields the macroscopic compliance tensor. The compliance tensor, in turn, is used to determine the lattice material's elastic moduli, Poisson's ratio and shear moduli.

When it comes to the forth type of assembly from Table 1, geometric nonlinearity comes into play and contributes to the comprehensive stiffness of the periodic framework. Kinematically indeterminate structures can gain rigidity under external loading or due to a pre-stressing of its members. When the internal mechanisms are classified as first-order infinitesimal mechanisms, which can be stiffened by states of self-stress, they form pre-stressed structures. In the following sections, we discuss the contribution of the geometrical stiffness to the comprehensive stiffness of tensegrity lattice materials to examine their effective behavior.

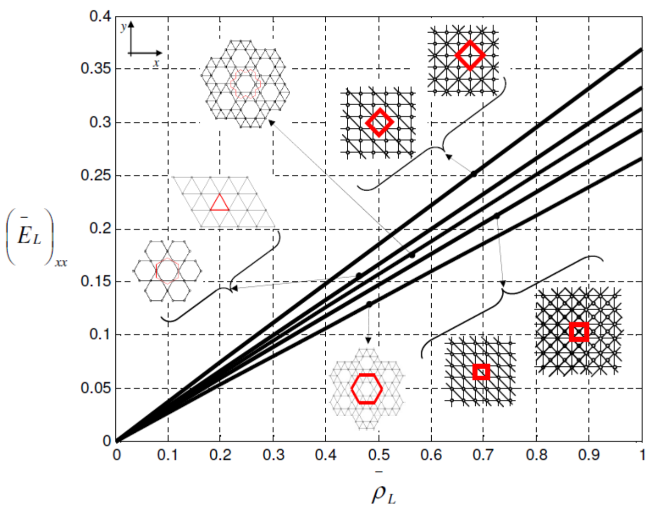

Figure 6 Relative Young's moduli in x-direction vs. relative density

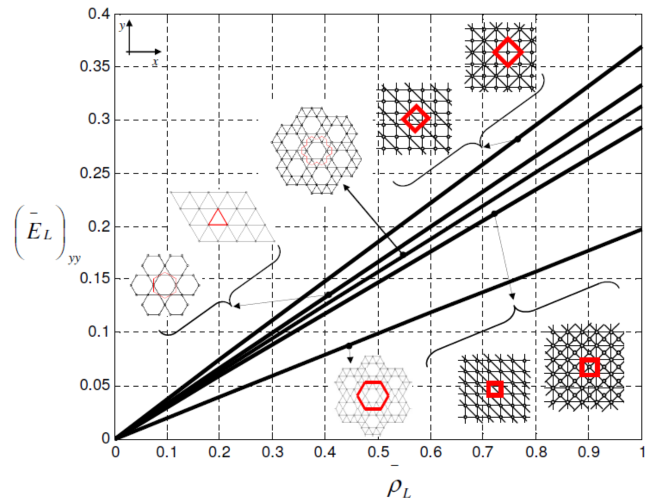

Figure 7 relative Young's moduli in y-direction vs. relative density

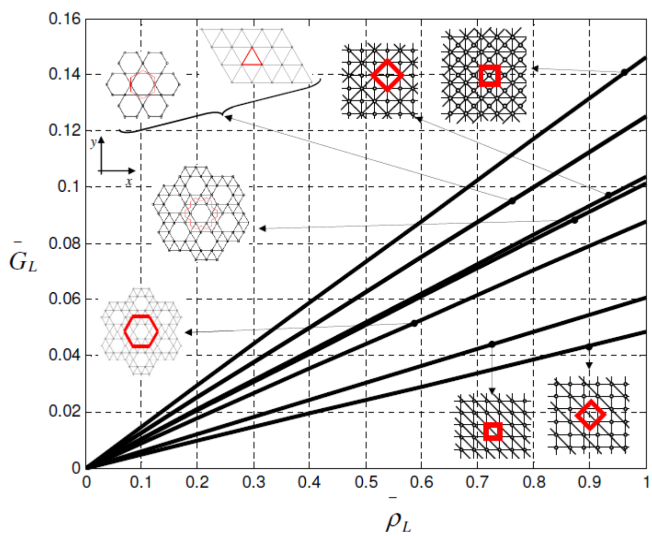

Figure 8 relative shear moduli vs. relative density

A. Periodicity-Induced Tensegrity Lattice Materials

The effective behavior of periodicity-induced tensegrity has been investigated by ElSayed [18] for pin- and rigid-jointed lattice materials. To determine the comprehensive stiffness of these statically and kinematically indeterminate structures, a summation of the material stiffness, K_E, and the geometrical stiffness, K_G, is necessary, so that, in general,

$$(K_E + K_G)d = f \tag{22}$$

and for pin-jointed frames (denoted by subscript p), we have

$$K_{C,p}d = \left(K_{E,p}^{bar} + K_{G,p}^{bar}\right)d = f \tag{23}$$

where $K_{C,p}, K_{E,p}^{bar}$ and $K_{G,p}^{bar}$ are the comprehensive, material and geometric stiffness of a pin-jointed frame composed of bars, respectively. To compute the comprehensive stiffness of an infinite periodic structure, first we need to formulate it for the unit cell finite structure. This is achieved by finding the comprehensive stiffness of each element in the unit cell and then assembling them into a global matrix. To account for the geometrical nonlinearity exhibited by a bar, k, the axial strain is found by

$$\epsilon_{x_1x_1} = \frac{\partial e_{kx_1}}{\partial x_1} + \frac{1}{2}\left(\frac{\partial e_{ky_1}}{\partial x_1}\right)^2 \tag{24}$$

where e_{kx_1} and e_{ky_1} are the axial and lateral linear displacements, respectively. The axial force in the bar is taken to be

$$P_k = \frac{AE}{l_k}(d_{jx_1} - d_{ix_1}) \tag{25}$$

where l_k is the length of the bar and d_{jx_1} and d_{ix_1} are the axial nodal deformations of nodes i and j of bar k. Castigliano's theorem is then used to obtain the nodal force-displacement relation, thus obtaining an expression that accounts for the material and geometrical stiffness of the bar

$$\left({}^{l}K_{E,p}^{bar} + {}^{l}K_{G,p}^{bar}\right)d_k^l = f_k^l \tag{26}$$

where the superscript l refers to the local coordinates along the axis of the bar. Transformation matrices are used in order to transform the bars from the local coordinate to the global coordinate of the unit cell. This allows for the assembly of a global matrix representing the unit cell comprehensive stiffness

$$K_{C,p}d = \left(K_{E,p}^{bar} + K_{G,p}^{bar}\right)d = f \tag{27}$$

Once the stiffness of the unit cell is obtained, it can be extended into the infinite periodic lattice using the Bloch theorem. The Cauchy-Born hypothesis is then used to homogenize the stiffness of the microscopic stiffness to obtain the macroscopic comprehensive stiffness of the lattice material. It employs transformation matrices, $\boldsymbol{M}$, that relate the nodal forces and deformations to the macroscopic strain field which, in turn, are used to derive the macroscopic stiffness of the pin-jointed lattice material [16]

$$\boldsymbol{K}_{E,p}^{bar} = \left(\frac{E}{2|Y|}\right)\left(\frac{\bar{\rho}}{C_T^{\rho}}\right)\left(\left({}^{f}\boldsymbol{M}_{E,p}^{bar}\right)^T\left({}^{d}\boldsymbol{M}_{E,p}^{bar}\right)\right) \tag{28}$$

$$\boldsymbol{K}_{G,p}^{bar} = \left(\frac{E}{2|Y|}\right)\left(\frac{\bar{\rho}}{C_T^{\rho}}\right)\left(\left({}^{f}\boldsymbol{M}_{G,p}^{bar}\right)^T\left({}^{d}\boldsymbol{M}_{G,p}^{bar}\right)\right) \tag{29}$$

where the superscripts f and d refer to the nodal forces and deformations respectively and $|Y|$ is the in-plane area of the unit cell. The elastic and shear moduli and Poisson's ratio are then determined. Examining the null space of the stiffness matrices allows for the classification of the lattice material. The number of macroscopic strain fields in the null space of the material, m_E, and comprehensive stiffness, m_C, classifies the lattice material into tensegrity, stretching-, and bending-dominated lattice material as shown in Table 2.

Table 2 Classification of lattice materials. Adapted from ElSayed [16]

Type	m_E	m_C	Class
I	= 0	= 0	Stretching-dominated
II	> 0	> 0	Bending-dominated
III	> 0	= 0	Tensegrity

Tensegrity lattice materials are unstable when examining their material stiffness alone and would collapse under macroscopic loading, however, taking into consideration their geometrical stiffness gives it rigidity. Unlike with tensegrity lattice materials, the influence the geometrical stiffness has on stretching-dominated lattice materials is negligible. This is demonstrated in the comparison of the elastic and shear moduli of the Kagome (classified as stretching-dominated) and $3^3 \cdot 4^2$ (named using the Schlafi system and classified as tensegrity) over a range of nominal strain shown in Figure 9.

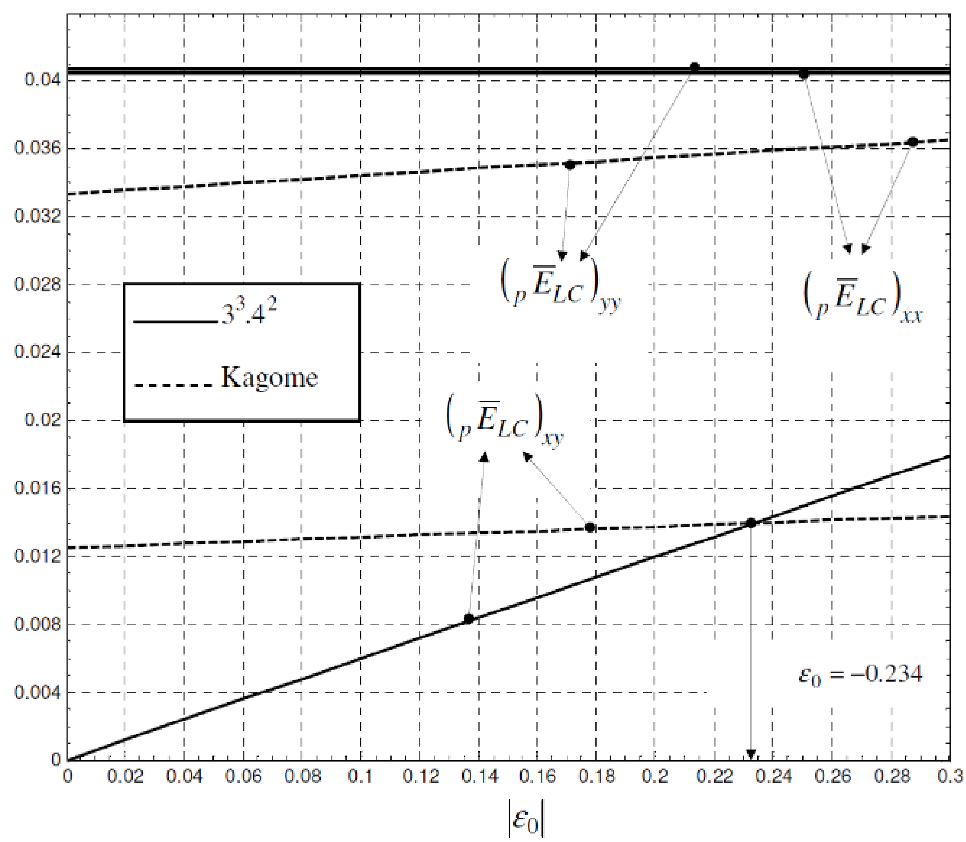

Figure 9 Elastic and shear moduli of the Kagome and $3^3 \cdot 4^2$ lattice materials over a range of nominal strain values

B. Stand-alone Tensegrity Lattice Materials

Next, we aim to study the effective behavior of stand-alone tensegrity of various topologies at different levels of pre-stress. We will be looking at planar [19] and spatial tensegrity cells, shown in Figure 10. To achieve variations in levels of pre-stress, we manipulate the lengths of the cables in the unit cell while keeping the distance between the two nodes they are connected to constant. In an effort to perform this characterization, we are using the same approach as with the periodicity-induced tensegrity which employs the Bloch theorem to express relations of the finite unit cell wave functions to the infinite periodic lattice and the Cauchy-Born Hypothesis to homogenize the microscopic stiffness properties to obtain the macroscopic effective behavior.

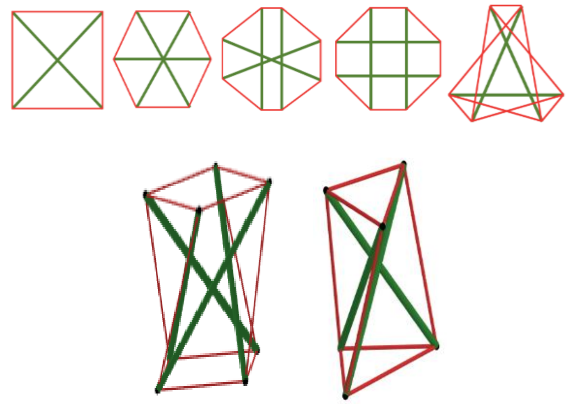

Figure 10 Topologies of stand-alone tensegrity elementary cells. Red lines refer to cables and green lines to struts

IV. Conclusion

Lattice materials are a rising candidate to achieve unprecedented properties of high strength and low density. The effect of the states of self-stress that may arise with the periodicity of a unit cell finite structure on the stiffening of first order infinitesimal mechanisms is explained. The homogenization method discussed in this paper employs the Bloch theorem and Cauchy-Born hypothesis to study the mechanical attributes of micro-lattice materials. We introduced tensegrity as the third class of lattice materials. The tensegrity class is then further subdivided into periodicity-induced and standalone tensegrity. It was revealed that the $3^3 \cdot 4^2$ topology is classified as a tensegrity lattice material whose geometrical stiffness influences its overall comprehensive stiffness significantly. The geometrical stiffness had negligible effect on the mechanical properties of the stretching-dominated Kagome lattice material. Periodicity induced lattice material offers promise in applications where large deformations are anticipated like in the skin panels of a morphing wing. The same analysis procedure is to be applied to stand alone tensegrity systems composed of bars and cables. The level of pre-stress experienced by this type of structure is expected to have an additional influence on the geometrical stiffness besides the periodicity of the lattice material.

Acknowledgements

The authors would like to acknowledge NSERC for the support and funding received through Discovery Grant RGPIN41735-2010

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